Confidence Intervals for the Autocorrelations of the Squares of GARCH Sequences

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Abstract. We compare three methods of constructing confidence intervals for sample autocorrelations of squared returns modeled by models from the GARCH family. We compare the residual bootstrap, block bootstrap and subsampling methods. The residual bootstrap based on the standard GARCH(1,1) model is seen to perform best.

1 Introduction

The paper is concerned with assessing finite sample performance of several methods of finding confidence intervals for autocorrelations of squared returns on speculative assets. While the returns themselves are essentially uncorrelated and most econometric and financial models explicitely imply that they are so, their squares exhibit a rich dependence structure. The autocorrelation function measures the strength of linear dependence and is a standard tool for assessing the impact of past information on the present.

We compare the performance of the various methods by means of their empirical coverage probability (ECP). Suppose we have a method of constructing, say, a 95% confidence interval $(\hat{l}_n, \hat{u}_n)$ from an observed realization $X_1, X_2, \ldots, X_n$. We simulate a large number R of realizations from a specific GARCH type model from which we construct R confidence intervals $(\hat{l}_n^{(r)}, \hat{u}_n^{(r)}),\ r = 1, 2, \ldots, R$. The percentage of these confidence intervals that contain the population autocorrelation is the ECP, which we want to be as close as possible to the nominal coverage probability of 95%.

Our objective is to provide answers to the following questions: Does any method have better ECP than the others? If not, what is the range of optimal applicability of each method? Is it better to use equal-tailed or symmetric confidence intervals (see Section 2.1)? How does the coverage depend on the value of γ_{c2} (see eq. (8))? For a given series length n, how should one choose the block length b for the block bootstrap and subsampling? For what lengths n do these methods yield useful confidence intervals?

The ultimate goal is to recommend a practical procedure for finding confidence intervals for squared autocorrelations which assumes minimal prior knowledge of the stochastic mechanism generating the returns.

In Section 2, we describe the three methods. Section 3 introduces the various GARCH models we use for the comparison. The results of our simulations are presented in Section 4 with broad conclusions summarised in Section 4.4.

For ease of reference, recall that the sample autocovariances of the squared returns are

$$\hat{\gamma}_{n,X^2}(h) = \frac{1}{n}\sum_{t=1}^{n-h}\left(X_t^2 - \frac{1}{n-h}\sum_{t=1}^{n-h}X_t^2\right)\left(X_{t+h}^2 - \frac{1}{n-h}\sum_{t=h+1}^{n}X_t^2\right) \quad (1)$$

whereas the population autocovariances are

$$\gamma_{X^2}(h) = E\left[(X_0^2 - EX_0^2)(X_h^2 - EX_0^2)\right]. \quad (2)$$

The corresponding autocorrelations are

$$\hat{\rho}_{n,X^2}(h) = \frac{\hat{\gamma}_{n,X^2}(h)}{\hat{\gamma}_{n,X^2}(0)}, \quad \rho_{X^2}(h) = \frac{\gamma_{X^2}(h)}{\gamma_{X^2}(0)}. \quad (3)$$

2 Confidence intervals for autocorrelations of squared returns

In this section we describe in detail the methods of constructing confidence intervals which we wish to compare. For concreteness, we focus on lag 1 autocorrelations, but the methods apply with minimal modifications (needed only for block bootstrap) to any lag.

2.1 Residual Bootstrap

To illustrate the idea and focus attention, we describe the method of residual bootstrap for the ARCH(1) model given by

$$X_t = \sigma_t Z_t, \quad \sigma_t^2 = \omega + \alpha X_{t-1}^2. \quad (4)$$

As can be gleaned from the exposition in Section 3, the method can be readily extended to any parametric model defined by GARCH type equations by computing the residuals $\hat{Z}_t = [\hat{\sigma}_t]^{-1} X_t$. Since the conditional volatility σ_t^2 is a function of the model parameters, past observations and past innovations, $\hat{\sigma}_t^2$ can be computed recursively once parameter estimates are available.

Assuming then that the observations $X_1, X_2, \ldots, X_n$ follow the ARCH(1) model (4), we proceed as follows:

1. Find estimates $\hat{\omega}$ and $\hat{\alpha}$ and form the residuals $\hat{Z}_t = [\hat{\omega} + \hat{\alpha} X_{t-1}^2]^{-1/2} X_t$, with suitably chosen X_0, e.g. the average of all X_t.
2. Form B bootstrap realizations $X_t^2(b) = [\hat{\omega} + \hat{\alpha} X_{t-1}^2(b)] \hat{Z}_t^2(b), \quad t = 1, 2, \ldots, n$, where $\hat{Z}_1^2(b), \ldots \hat{Z}_n^2(b),\ b = 1, 2, \ldots, B$, are the B bootstrap samples selected with replacement from the squared residuals $\hat{Z}_1^2, \ldots \hat{Z}_n^2$.
3. Calculate the bootstrap autocorrelations $\rho_{n,X^2}^{(b)}(1),\ b = 1, 2, \ldots, B$ and use their empirical quantiles to find a confidence interval for $\rho_{n,X^2}(1)$.

In step 1), we use quasi maximum likelihood estimators (QMLE's) of model parameters which maximize the likelihood function computed under the assumption that the innovations Z_t are standard normal.

We now enlarge on step 3). Denote by $F_{\rho(1)}^*$ the EDF (empirical distribution function) of the $\rho_{n,X^2}^{(b)}(1),\ b = 1, 2, \ldots, B$. The $(\alpha/2)$th and $(1-\alpha/2)$th quantiles of $F_{\rho(1)}^*$ will yield an *equal-tailed* $(1-\alpha)$ level confidence interval. To construct a *symmetric* confidence interval centered at $\hat{\rho}_{n,X^2}(1)$, we need the empirical distribution $F_{\rho(1),|\cdot|}^*$ of the B values $|\rho_{n,X^2}^{(b)}(1) - \hat{\rho}_{n,X^2}(1)|$. Denote by $q_{|\cdot|}(1-\alpha)$ the $(1-\alpha)$ quantile of $F_{\rho(1),|\cdot|}^*$. Then the symmetric confidence interval is

$$\left(\hat{\rho}_{n,X^2}(1) - q_{|\cdot|}(1-\alpha), \quad \hat{\rho}_{n,X^2}(1) + q_{|\cdot|}(1-\alpha)\right).$$

A usual criticism of methods based on a parametric model is that misspecification can lead to large biases. In many applications however these biases have only negligible impact on a statistical procedure of interest. In our setting, it may well be the case that the residual bootstrap confidence intervals based on a misspecified model can produce good coverage probabilities. This point is taken up again after empirical evidence has been presented.

2.2 Block bootstrap

In this section we describe how the popular block-bootstrap of [6] can be used to construct confidence intervals for autocorrelations. This method does not require a model specification, but it relies on a choice of the block size b which is often a difficult task. A good account of block bootstrap is given in [2].

Focusing again on lag one sample autocorrelation of the squared observations, we proceed as follows:

Having observed the sample $X_1^2, \ldots, X_n^2$ form the vectors

$$\mathbf{Y}_2 = [X_1^2, X_2^2]', \mathbf{Y}_3 = [X_2^2, X_3^2]', \ldots, \mathbf{Y}_n = [X_{n-1}^2, X_n^2]'.$$

There are $n-1$ such vectors. Now choose a block length b and compute the number of blocks $k = [(n-1)/b] + 1$ (if $(n-1)/b$ is an integer we take $k = (n-1)/b$). Choose k blocks with replacement to obtain kb vectors. Choosing the k blocks corresponds to generating k observations from the uniform distribution on $\{2, 3, \ldots, n-b+1\}$. Denote these observations $j_1, j_2, \ldots, j_k$. We thus obtained the kb vectors

$$\mathbf{Y}_{j_1}, \mathbf{Y}_{j_1+1}, \ldots, \mathbf{Y}_{j_1+b-1}, \ldots, \mathbf{Y}_{j_k}, \mathbf{Y}_{j_k+1}, \ldots, \mathbf{Y}_{k_1+b-1}.$$

If $(n-1)/b$ is not an integer, remove the last few vectors to have exactly $n-1$ vectors. This gives us the bootstrap vector process

$$\mathbf{Y}_2^* = [X_1^{*2}, X_2^{*2}]', \mathbf{Y}_3^* = [X_2^{*2}, X_3^{*2}]', \ldots, \mathbf{Y}_n^* = [X_{n-1}^{*2}, X_n^{*2}]'.$$

The bootstrap sample autocovariances are computed according to (1) with the X_t replaced by the X_t^* defined above. The empirical distribution of $\hat{\rho}^*_{n,X^2}(1)$ is then an approximation to the distribution of $\hat{\rho}_{n,X^2}(1)$. As described in Section 2.1, the quantiles of the empirical distribution of $|\hat{\rho}^*_{n,X^2}(1) - \hat{\rho}_{n,X^2}(1)|$ can be used to construct symmetric confidence intervals.

2.3 Subsampling

The subsampling methodology is described in detail in [7]. [9] investigated subsampling confidence intervals for autocorrelations of linear time series models like ARMA. In this section we adapt their methodology to the squares of GARCH processes.

To lighten the notation, denote $U_t = X_t^2 - \frac{1}{n}\sum_{j=1}^n X_j^2$ and suppress the subscript X^2 in the following formulas in which use definitions (1) and (3). Set

$$s_n^2(h) = \frac{1}{n}\sum_{j=1}^{n-h}\left(U_{j+h} - \hat{\rho}_n(h)U_j\right)^2, \quad \hat{\sigma}_n^2(h) = \frac{s_n^2(h)}{\sum_{j=h}^n U_j^2} \tag{5}$$

and consider the studentized statistic

$$\hat{\xi}_n = \frac{\hat{\rho}_n(h) - \rho_n(h)}{\hat{\sigma}_n(h)}. \tag{6}$$

To construct equal-tailed and symmetric confidence intervals, we would need to know the sampling distribution of $\hat{\xi}_n$ and $|\hat{\xi}_n|$, respectively. We use subsampling to approximate these distributions: Consider an integer $b < n$ and the $n - b + 1$ blocks of data $X_t, \ldots, X_{t+b-1}$, $t = 1, \ldots, n-b+1$. From each of these blocks compute $\hat{\rho}_{b,t}(h)$ and $\hat{\sigma}_{b,t}(h)$ according to respectively (1), (3) and (5), but replacing the original data $X_1, \ldots, X_n$ by $X_t, \ldots, X_{t+b-1}$. Next, compute the subsampling counterpart of the studentized statistic (6) $\hat{\xi}_{b,t}(h) = \dfrac{\hat{\rho}_{b,t}(h) - \hat{\rho}_n(h)}{\hat{\sigma}_{b,t}(h)}$ and construct the empirical distribution functions

$$L_b(x) = \mathcal{N}_b^{-1}\sum_{t=1}^{n-b+1} \mathbf{1}\left\{\hat{\xi}_{b,t}(h) \le x\right\}, \quad L_{b,|\cdot|}(x) = \mathcal{N}_b^{-1}\sum_{t=1}^{n-b+1} \mathbf{1}\left\{|\hat{\xi}_{b,t}(h)| \le x\right\},$$

with $\mathcal{N}_b = n-b+1$. The empirical quantiles of L_b and $L_{b,|\cdot|}$ allow us to construct, respectively, equal-tailed and symmetric confidence intervals. For example, denoting by $q_{b,|\cdot|}(1-\alpha)$ the $(1-\alpha)$th quantile of $L_{b,|\cdot|}$, a subsampling symmetric $1-\alpha$ level confidence interval for $\rho_n(h)$ is

$$\left(\hat{\rho}_n(h) - \hat{\sigma}_n(h) q_{b,|\cdot|}(1-\alpha), \quad \hat{\rho}_n(h) + \hat{\sigma}_n(h) q_{b,|\cdot|}(1-\alpha)\right).$$

3 GARCH models

In this Section we describe the GARCH models which we used to assess the performance of confidence interval building procedures. in a uniform framework, it is convenient to consider a general class of GARCH(1,1) type models studied by [5]. The observations X_t are thus assumed to satisfy $X_t = Z_t\sigma_t$, where Z_t is a sequence of independent identically distributed random variables with zero mean and

$$\sigma_t^2 = g(Z_{t-1}) + c(Z_{t-1})\sigma_{t-1}^2 \tag{7}$$

We considered only specifications (7) with in which the function $g(\cdot)$ is a constant and the Z_t are standard normal. Denoting $\gamma_{ci} = Ec^i(Z_t)$, [5] proved that under the above assumptions a sufficient and necessary condition for the existence of the $2m$th unconditional moment of X_t is $\gamma_{cm} = Ec_t^m < 1$. Thus, the fourth unconditional moment of X_t exits if and only if

$$\gamma_{c2} = Ec_t^2 \in [0,1). \tag{8}$$

We considered the following three specific models:

1. The standard GARCH$(1,1)$ model, see [1]; [10], pp. 78-79) for which

$$c_{t-1} = \beta + \alpha Z_{t-1}^2, \quad \sigma_t^2 = \omega + \alpha X_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{9}$$

2. The GJR-GARCH$(1,1)$ model, see [4], with

$$c_{t-1} = \beta + (\alpha + \phi I(Z_{t-1}))Z_{t-1}^2, \quad \sigma_t^2 = \omega + (\alpha + \phi I(Z_{t-1}))X_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{10}$$

   where $I(Z_{t-1}) = 1$ if $Z_{t-1} < 0$, and $I(Z_{t-1}) = 0$ otherwise.
3. The nonlinear GARCH(1,1) model (NLGARCH(1,1,2), see [3], with

$$c_{t-1} = \beta + \alpha(1 - 2\eta\ \text{sign}(Z_{t-1}) + \eta^2)Z_{t-1}^2;$$

$$\sigma_t^2 = \omega + \alpha(1 - 2\eta\ \text{sign}(Z_{t-1}) + \eta^2)X_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{11}$$

Assuming the errors Z_t have standard normal distribution, the values of γ_{c2} and $\rho_{X^2}(1)$ can be computed in a closed form.The expressions are however complex and are not given here. What is relevant, is that if we know the model parameters, we can calculate precisely the population autocorrelation $\rho_{X^2}(1)$ and the value of γ_{c2}.

For each of the three models, we considered five parameter choices, which we labeled as models 1 through 5. The lag one autocorrelations for these choices are, respectively, approximately .15, .22, .31, .4, .5. The corresponding values of γ_{c2} are respectively, approximately .1, .3, .5, .7, .9. To facilitate comparison, models with the same index have similar values of γ_{c2} and $\rho_{X^2}(1)$, eg standard GARCH and GJR-GARCH with index 3 both have $\gamma_{c2} \approx .5$ and $\rho_{X^2}(1) \approx .31$.

4 Simulation results

The objective of the simulations presented in this section was to investigate the performance of the three methods described in Section 2. This is done by comparing the empirical coverage probabilities (ECP's) for the fifteen data generating processes (DGP's) introduced in Section 3. We generated one thousand replications of each DGP and considered realizations of length $n = 100, 250, 500, 1000$. We focused on the most commonly used confidence level of 95%. The standard errors in all tables are about 0.5% and are always smaller than 1%.

We first examine in Sections 4.1–4.3 each method separately what allows us to focus on specific issues pertaining to each method. We then combine and summarize our findings in Section 4.4.

4.1 Residual Bootstrap

Table 1. Empirical coverage probabilities of *symmetric* confidence intervals constructed using *residual bootstrap.*

n	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)
STD GARCH	1	2	3	4	5
100	99.6	85.3	86.0	80.4	77.4
250	92.9	91.3	92.1	89.4	84.4
500	93.4	93.4	94.1	93.7	92.7
1000	95.1	96.8	97.6	97.6	94.4
GJR GARCH	1	2	3	4	5
100	97.7	94.8	92.0	89.5	81.5
250	96.2	96.6	97.0	96.4	92.3
500	98.3	99.2	98.9	99.1	96.5
1000	99.0	99.4	99.6	99.8	98.8
NL GARCH	1	2	3	4	5
100	95.5	83.8	79.8	74.7	66.0
250	91.7	87.3	84.3	81.0	73.6
500	91.7	93.1	88.5	82.1	77.3
1000	96.4	93.3	92.9	87.0	81.0

Table 1 presents the empirical coverage probabilities of the symmetric confidence interval for the three GARCH models. To save space the results for the equal-tailed confidence interval are not presented, but are discussed in the following conclusions.

Equal-tailed and symmetric confidence intervals perform equally well for the standard GARCH and GJR-GARCH. However, for the NLGARCH, the symmetric interval is better than the equal-tailed. It is thus seen that the symmetric confidence interval is preferred over the equal-tailed. The ECP decreases as the value of γ_{c2} approaches 1. Recall that $\gamma_{c2} < 1$ is required for the population autocovariances to exist. When $\gamma_{c2} \approx 0.9$, at least 250 observations are needed to ensure reasonable ECP for the standard GARCH and the GJR-GARCH. For the NL GARCH, even series length of 1000, does not produce satisfactory results. For the standard GARCH and the GJR-GARCH increasing the sample size from 500 to 1000 does not improve the ECP. For the NL GARCH a sample size of 1000 observations is needed, except when $\gamma_{c2} \leq 0.3$.

The somewhat worse performance of the residual bootstrap method for the GJR-GARCH which becomes markedly worse for the NL GARCH can be attributed to identification problems, which are particularly acute for the NL GARCH. We noticed that for the latter model biases of parameter estimates are very large when η in equation (11) is large. Large η corresponds to large γ_{c2}, we omit the details of the calculation. On the other hand, for the standard GARCH, while they still do exist, the identification problems are much less severe. It is therefore worthwhile to investigate if estimating the standard GARCH model on all three DGP's might lead to improvements in ECP's. Figure 1 shows that this in fact the case for symmetric confidence intervals and series length 500. The results for other series lengths look very much the same and are therefore not presented.

In conclusion, the residual bootstrap method works best if symmetric confidence intervals are used and the standard GARCH model is estimated. Thus, in our context, misspecifying a model improves the performance of the procedure.

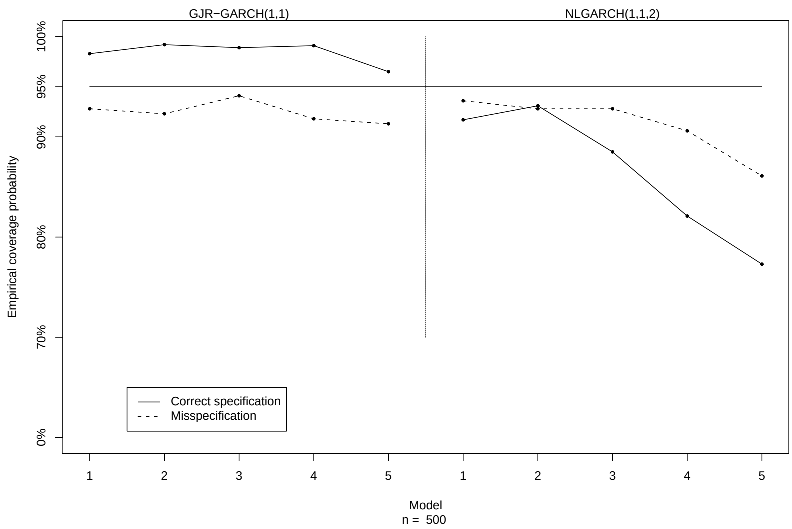

Fig. 1. Comparison of ECP's for symmetric residual bootstrap confidence intervals based on standard GARCH and a correct specification. The nominal coverage of 95% is marked by the solid horizontal line. The series length is $n = 500$.

4.2 Block bootstrap

The implementation of this method requires a choice of the block length b. We therefore have a multitude of cases to explore: 15 models, 2 types of confidence intervals (equal-tailed and symmetric), 4 sample sizes and several choices of b. Since we used 10 values of b in our experiments, we obtained 1,200 ECP's. Presenting all these results in tables would take up too much space, so we restrict ourselves to describing them and presenting some typical values in Table 2.

From our simulations we draw the following conclusions. The empirical coverage probabilities are generally too low for all choices of n and b and are in the range of 80% to 90% for $\gamma_{c2} \leq 0.3$ and go down to slightly above 50% for $\gamma_{c2} \approx 0.9$. Irrespective of the value of γ_{c2}, choosing smaller b gives higher coverage. However, extremely small b, like 1 or 2, do not work well. We recommend to use $b = 3$ or $b = 5$. The dependence on b is however not substantial, which is very desirable, as in many other applications choosing optimal b is very difficult. There is not much difference of ECPs between equal-tailed and symmetric confidence intervals. The block bootstrap confidence intervals are generally too short and given that the QML estimates underestimate the true value of the autocorrelation, they are shifted too much to the left what causes the under-coverage.

Table 2. Empirical coverage probabilities of *symmetric* confidence intervals based on the *block bootstrap* method for the five parameter choices in the *GJR-GARCH* model.

Model		1	2	3	4	5
n	b	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)
500	3	87.0	82.0	78.4	65.5	61.4
	5	89.1	83.8	73.4	63.0	58.5
	10	87.9	81.8	71.4	60.6	51.9
	15	84.5	78.7	71.8	63.8	52.7
	20	84.6	81.0	71.1	62.7	51.5
	30	85.6	79.0	69.6	61.3	50.0
	40	85.3	79.3	67.1	58.7	48.7
1000	5	87.7	84.4	75.2	67.9	59.6
	10	88.6	85.1	70.8	61.0	52.6
	15	89.7	83.0	72.7	63.6	53.3
	20	88.6	83.9	73.9	60.6	51.2
	30	87.8	80.9	72.7	59.7	51.2
	40	86.1	82.2	72.6	60.8	52.8
	50	85.8	82.4	69.9	59.5	50.6

4.3 Subsampling

This method also requires the block length b. Our simulations led us to the following conclusions:

The subsampling method is very sensitive to the choice of b. Symmetric confidence intervals have a much better ECP than the equal-tailed. By choosing very short b's, such as 3 or 6, we can obtain ECP's that are quite close to 95% for models with $\gamma_{c2} < 0.6$ and fair coverage for models with greater values of γ_{c2}. Such choice of b is somewhat surprising, as autocovariances are then computed from very short subseries. The ECP's are generally too low for equal-tailed confidence intervals and are typically in the range of 50-70%. As γ_{c2} approaches 1, the empirical coverage decrease and in some cases may be as low as 10%.

Typical ECP's are shown in Table 3.

Table 3. Empirical coverage probabilities of *symmetric* confidence interval based on the *subsampling* method for the five parameter choices in the *NLGARCH* model. Series length $n = 500$.

Model	1	2	3	4	5
b	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)	*e.c.p.* (%)
3	97.2	95.3	91.6	82.3	70.4
6	94.1	95.5	79.9	67.9	51.5
8	90.1	83.0	75.1	63.3	50.2
10	85.4	80.9	71.4	57.5	44.5
50	80.2	76.1	63.9	54.1	41.2

4.4 Conclusions and practical recommendations

Our simulations show that the best method is residual bootstrap implemented assuming a standard GARCH(1,1) model as discussed towards the end of Section 4.1.

The block bootstrap and subsampling methods do not perform well when γ_{c2} approaches 1. Moreover, these methods require a choice of the block size b. The latter problem is particularly acute for the subsampling method. Except for the NL-GARCH, the residual bootstrap method with correct model specification performs reasonably well even for γ_{c2} close to 1. This is probably due to the fact that large values of γ_{c2} correspond to large values of model parameters which are easier to estimate than small values yielding residuals which are close to the unobservable errors.

To illustrate our finding we present in Figure 2 a graphical comparison of symmetric confidence intervals based on the three methods for $n = 1000$. Graphs for other values of n are similar. For the block bootstrap and subsampling methods we used values of b which give optimal ECP's for most models. The residual bootstrap is based on correct specification.

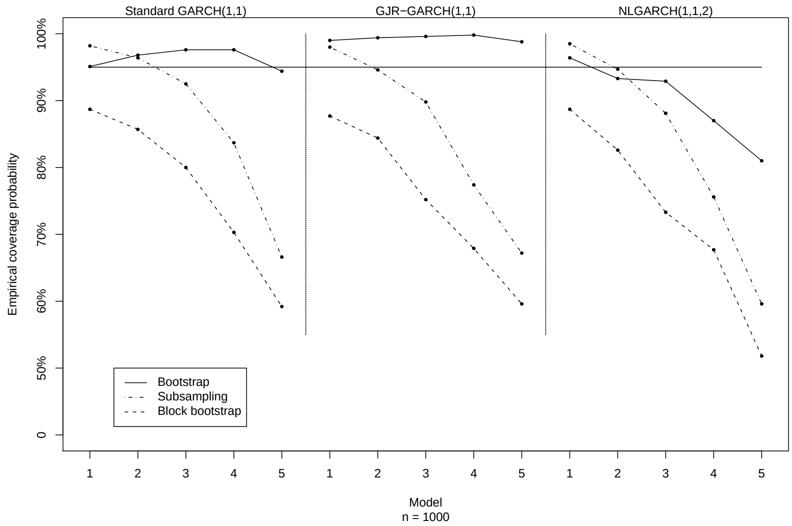

Fig. 2. Comparison of ECP's for symmetric confidence intervals. The nominal coverage 95% is marked by solid horizontal line. The series length is $n = 1000$. For block bootstrap, $b = 5$, for subsampling $b = 3$.

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